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# 이벤트 트리거 ϵ -PID 제어기를 통한 DC 모터 제어

# Event-triggered ϵ -PID Control of a DC Motor

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**Abstract:** In this study, an event-triggered ϵ -PID controller has been proposed for controlling the position of a DC motor. An event-triggering condition with a pre-defined positive constant T was designed by modifying the framework of the event-triggered control method to prevent the Zeno behavior. The controlled system was analyzed using the Lyapunov and Razumikhin stability theorems. For comparison, simulation results have also been provided that further highlight and confirm the merit of the proposed control method over the conventional time-triggered control technique.

**Keywords:** DC motor, event-triggered control, ϵ -PID control

## I. INTRODUCTION

In [1], an $\epsilon$-PID controller is suggested for the position control of a DC motor where the gain-scaling parameter $\epsilon$ is effectively used in tuning control parameters for adjusting control performance. On the other hand, there have been a number of results on event-triggered control methods where the main merit is the efficient usage of communication resources due to the discretely updated control inputs. In [2], the design of event-triggered non-fragile dynamic output feedback controllers for linear systems facing actuator saturation and disturbances is addressed, incorporating additive gain variations. In [3], they introduce a control-based event-triggered sliding mode control for networked linear systems, focusing on triggering events based on deviations in control input rather than system states. In [4], various event-triggered schemes, including sampling and communication methods, are addressed. This paper explores four key models for event-triggered control systems and summarizes methods for stability analysis and controller design. In [5], the conditions for achieving consensus both with and without external disturbances are established, ensuring that Zeno behavior is avoided. In [6], an input-to-state stability property for the closed-loop system is established through cooperative systems theory. The proposed method is demonstrated through its application to a gyroscopic control problem in marine robotics for curve tracking dynamics. In [7], they derive LMI-based conditions for computing controller and observer gains. In addition, as a study on DC motor control, the paper [12] proposed a recurrent neural network-based inverse model controller and applied it to the current controller of a DC motor coupled with a reducer.

Note that the event-triggered control methods have certain merits as mentioned above, yet at the same time, they have some disadvantages such as complicated analysis related with triggering conditions and somewhat difficult implementation due to memory requirement [11]. In this regard, in [8], a zero-order-hold (ZOH) based event-triggered control (ETC) method is proposed where its merits lie in two aspects: simplified system analysis regarding the Zeno behavior and even less number of control inputs updates compared to the traditional event-triggered control methods.

In this paper, we newly suggest an event-triggered $\epsilon$ -PID controller for the position control of a DC motor by following the ZOH-based ETC method of [8] where the merit of using gain-scaling parameter $\epsilon$ still remains valid and there is an additional merit of using less control input updates due to intrinsic nature of the event-triggered control method. Moreover, due to the ZOH feature, the additional analysis regarding the Zeno behavior is not needed. With our proposed control scheme, we carry out the rigorous system analysis by utilizing both Lyapunov and Razumikhin theorem and show that the controlled system is well bounded whose ultimate bounds can be manipulated by using $\epsilon$. Our controller is proposed for the position control of a DC motor where the gain-scaling parameter ϵ is utilized to fine-tune the control parameters allowing for adjustments in control performance over [6] and [8]. Then, we perform the numerical simulation in order to illustrate the merits of our proposed control method over the traditional time-triggered control method [1] by comparison.

## II. SYSTEM AND PROBLEM FORMULATION

The dynamic equation of a (MAXON) DC motor is given by [1]

$$0 = \frac{K_m}{rR}u - \frac{J_m}{r^2}\ddot{q} - \frac{B_m + K_b K_m/R}{r^2}\dot{q} - w(t) \quad (1)$$

where $q$ is the position link, $K_m$ is the torque constant, $J_m$ is the motor inertia, $B_m$ is the damping coefficient, $K_b$ is the back emf constant, $R$ is the armature resistance, $r$ is the fear ratio, $w(t)$ is the load torque, and $u$ is the control input in voltage.

Let $x_1 = q, x_2 = \dot{q}, e_1 = x_1 - q_d$, and $e_2 = x_2$ where $q_d$ is a piecewise constant signal. Then, the system (1) turns into

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = -ae_2 + bu - cw(t) \quad (2)$$

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where

$$a = \frac{B_m + K_b K_m / R}{J_m}, \quad b = \frac{r K_m}{J_m R}, \quad c = \frac{r^2}{J_m} \tag{3}$$

Thus, the considered system (2) represented in a state-space equation is a linear system with a nonvanishing disturbance signal. Considering the usual cases in practical applications, $w(t)$ is assumed to be finitely bounded for all $t \geq 0$. Our control goal is to design an event-triggered PID controller to make $q$ track a bounded signal $q_d$.

## III. EVENT-TRIGGERED $\epsilon$-PID CONTROLLER AND SYSTEM ANALYSIS

By adding an integrator as $\dot{e}_0 = e_1$ to the system (2), we obtain a third-order system as

$$\dot{e} = Ae + B[-ae_2 + bu - cw(t)] \tag{4}$$

where $e = [e_0, e_1, e_2]^T$ and $(A, B)$ is a Brunovsky canonical pair.

By considering (4), we can give a time-triggered $\epsilon$-PID controller [1] as

$$\begin{aligned} \bar{u} &= \frac{k_1}{b\epsilon^3} e_0 + \frac{k_2}{b\epsilon^2} e_1 + \left(\frac{k_3}{b\epsilon} + \frac{a}{b}\right) e_2 \\ &= K_P(\epsilon) e_1 + K_I(\epsilon) \int_0^t e_1(s) ds + K_D(\epsilon) \dot{e}_1 \end{aligned} \tag{5}$$

where

$$K_P(\epsilon) = \frac{k_2}{b\epsilon^2}, \quad K_I(\epsilon) = \frac{k_1}{b\epsilon^3}, \quad K_D(\epsilon) = \frac{k_3}{b\epsilon} + \frac{a}{b} \tag{6}$$

with $0 < \epsilon < \infty$ to be chosen.

By utilizing (5), we propose the following event-triggered $\epsilon$-PID controller

$$u = \bar{u}(t_i), \quad \forall t \in [t_i, t_{i+1}) \tag{7}$$

with a triggering condition of

$$\begin{aligned} t_{i+1}^{*} &= \inf\{t > t_i : |\bar{u} - u| > \sigma \|E_\epsilon e\|\} \\ t_{i+1}^{**} &= T + t_i \\ t_{i+1} &= \max\{t_{i+1}^{*}, t_{i+1}^{**}\} \end{aligned} \tag{8}$$

where $t_0 = 0, i = 0,1,\cdots, E_\epsilon = \mathrm{diag}[1, \epsilon, \epsilon^2]$, and the parameters $0 < \sigma, T < 1$ are to be chosen.

***System Analysis:*** From (4) and (7), for $t \in [t_i, t_{i+1})$, the closed-loop system is

$$\dot{e} = A_{K(\epsilon)} e + B \cdot b(u - \bar{u}) - B \cdot cw(t) \tag{9}$$

where $A_{K(\epsilon)} = A + BK(\epsilon)$ with $K(\epsilon) = [k_1/\epsilon^3, k_2/\epsilon^2, k_3/\epsilon]$.

Here, we consider two cases as follows

Case 1. $t_{i+1} = t_{i+1}^{*}$.

Following [1], by assuming that $A_K = A_{K(1)}$ is Hurwitz, we can use a Lyapunov equation of $A_{K(\epsilon)}^T P_\epsilon + P_\epsilon A_{K(\epsilon)} = -\epsilon^{-1} E_\epsilon^2$ where $P_\epsilon = E_\epsilon P E_\epsilon$ and $A_K^T P + P A_K = -I$. Then, with a Lyapunov function of $V(e) = e^T P_\epsilon e$, along the trajectory of (9),

$$\begin{aligned} \dot{V}(e) &= -\epsilon^{-1} \|E_\epsilon e\|^2 + 2e^T P_\epsilon [B \cdot b(u - \bar{u}) - B \cdot cw(t)] \\ &\leq -\epsilon^{-1} \|E_\epsilon e\|^2 + 2b\|P\| \|E_\epsilon e\| \|E_\epsilon B(u - \bar{u})\| \\ &\quad + 2c\|P\| \|E_\epsilon e\| \|E_\epsilon B w(t)\| \end{aligned} \tag{10}$$

Note that, for $t \in [t_i, t_{i+1})$, $|u - \bar{u}| \leq \sigma \|E_\epsilon e\|$ and there exists a finite constant $\overline{w}$ such that $w(t) \leq \overline{w}$ for all $t \geq 0$. So, inserting these inequalities into (10), we have

$$\begin{aligned} \dot{V}(e) \leq & -\frac{\epsilon^{-1}}{2}(1 - 4b\|P\|\epsilon^3 \sigma) \|E_\epsilon e\|^2 \\ & -\frac{\epsilon^{-1}}{2} \|E_\epsilon e\| (\|E_\epsilon e\| - 4c\|P\| \overline{w} \epsilon^3) \end{aligned} \tag{11}$$

From (11), we can see that when we select $\sigma$ and $\epsilon$ such that $1 - 4b\|P\|\epsilon^3 \sigma > 0$, $\|E_\epsilon e\|$ is ultimately bounded by $O(\epsilon^3)$ [9], which means that the ultimate bound of $\|e\|$ becomes smaller as $\epsilon$ decreases.

Case 2. $t_{i+1} = t_{i+1}^{**}$.

In this case, we need to investigate the upper bound of $u - \bar{u}$ differently from Case 1. First, we have

$$\begin{aligned} u - \bar{u} &= \frac{1}{b} K(\epsilon)[e(t_i) - e] + \frac{a}{b}[e_2(t_i) - e_2] \\ &= -\frac{1}{b} K(\epsilon) \int_{t_i}^t \dot{e}(s) ds - \frac{a}{b} \int_{t_i}^t \dot{e}_2(s) ds \end{aligned} \tag{12}$$

Here, note that

$$\dot{e} = Ae + B\dot{e}_2 \tag{13}$$

$$\dot{e}_2 = K(\epsilon) e(t_i) + a[e_2(t_i) - e_2] - cw(t) \tag{14}$$

By inserting (13) and (14) into (12), we can derive the inequality as follows.

$$\begin{aligned} u - \bar{u} = & -\frac{1}{b} K(\epsilon) A E_\epsilon^{-1} \int_{t_i}^t E_\epsilon e(s) ds \\ & -\frac{1}{b} K(\epsilon) B K(\epsilon) E_\epsilon^{-1} \int_{t_i}^t E_\epsilon e(t_i) ds \\ & -\frac{a}{b} K(\epsilon) B \epsilon^{-2} \int_{t_i}^t \epsilon^2 [e_2(t_i) - e_2(s)] ds \\ & +\frac{c}{b} K(\epsilon) B \int_{t_i}^t w(s) ds - \frac{a}{b} K(\epsilon) E_\epsilon^{-1} \int_{t_i}^t E_\epsilon e(t_i) ds \\ & -\frac{a^2}{b} \epsilon^{-2} \int_{t_i}^t \epsilon^2 [e_2(t_i) - e_2(s)] ds + \frac{ac}{b} B \int_{t_i}^t w(s) ds \\ \leq & \frac{1}{b} \|K(\epsilon) A E_\epsilon^{-1}\| \cdot T \|E_\epsilon e_t\| + \frac{1}{b} \|K(\epsilon) B K(\epsilon) E_\epsilon^{-1}\| \cdot T \|E_\epsilon e_t\| \\ & + \frac{a}{b} \|K(\epsilon) B \epsilon^{-2}\| \cdot 2T \|E_\epsilon e_t\| + \frac{c}{b} \|K(\epsilon) B\| \cdot T \overline{w} \\ & + \frac{a}{b} \|K(\epsilon) E_\epsilon^{-1}\| \cdot T \|E_\epsilon e_t\| + \frac{a^2}{b} \epsilon^{-2} \cdot 2T \|E_\epsilon e_t\| \\ & + \frac{ac}{b} \cdot T \overline{w} \end{aligned} \tag{15}$$

Where $\|E_\epsilon e_t\| = \sup_{-T \leq \theta \leq 0} \|E_\epsilon e(t + \theta)\|$.

We use a Razumikhin theorem [10] to deal with a term $\|E_\epsilon e_t\|$. Thus, from $V(e_t) < qV(e)$, $q > 1$, we can obtain that $\|E_\epsilon e_t\| < \bar{q} \|E_\epsilon e\|$ where $\bar{q} = q\sqrt{\lambda_{max}(P)/\lambda_{min}(P)}$. Thus, (15) becomes

$$|u - \bar{u}| \leq \Delta_1(\epsilon, T) \|E_\epsilon e\| + \Delta_2(\epsilon, T) \overline{w} \tag{16}$$

Where

$$\begin{aligned} \Delta_1(\epsilon, T) = \bar{q} \Big( & \frac{1}{b} \|K(\epsilon) A E_\epsilon^{-1}\| + \frac{1}{b} \|K(\epsilon) B K(\epsilon) E_\epsilon^{-1}\| + \\ & \frac{2a}{b} \|K(\epsilon) B \epsilon^{-2}\| + \frac{a}{b} \|K(\epsilon) E_\epsilon^{-1}\| + \frac{2a^2}{b} \epsilon^{-2} \Big) T \end{aligned} \tag{17}$$

$$\Delta_2(\epsilon, T) = \left(\frac{c}{b} \|K(\epsilon) B\| + \frac{ac}{b}\right) T \tag{18}$$

Now, from (10) and (16), noting that $\|E_\epsilon B\| = \epsilon^2$, it is easy to get

$$\begin{aligned} \dot{V}(e) \leq & -\frac{\epsilon^{-1}}{2} \left(1 - 4b\|P\|\epsilon^3 \Delta_1(\epsilon, T)\right) \|E_\epsilon e\|^2 \\ & -\frac{\epsilon^{-1}}{2} \|E_\epsilon e\| (\|E_\epsilon e\| - 4\|P\| [b\Delta_2(\epsilon, T) + c] \epsilon^3 \overline{w}) \end{aligned} \tag{19}$$

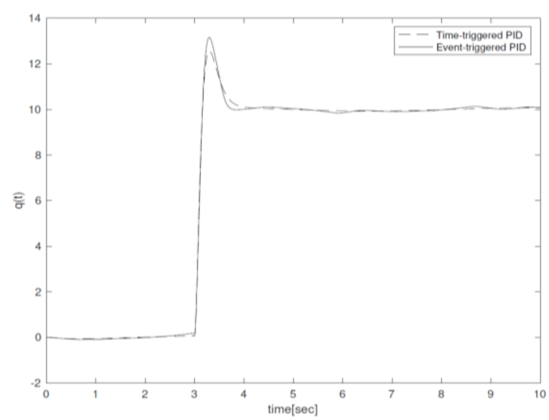
From (19), we can see that when we select $\epsilon$ and $T$ such that

$1-4b\|P\|\epsilon^2\Delta_1(\epsilon,T)>0$, $\|E_\epsilon e\|$ is ultimately bounded by $O(\epsilon^3)$ [9], which means that the ultimate bound of $\|e\|$ becomes smaller as $\epsilon$ decreases.

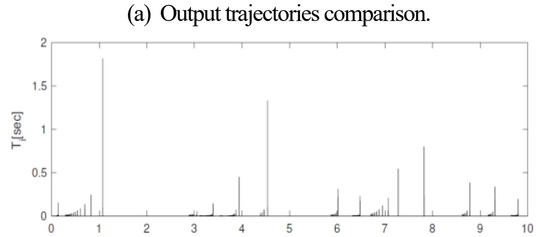
*Remark* 1. Due to the presence of T from (8), the positive lower bounds of the interexecution times $t_{i+1}-t_i \geq T > 0$ is guaranteed by default. So, any additional analysis related to the Zeno behavior is not needed.

## IV. SIMULATION RESULTS

We apply our proposed event-triggered ϵ-PID controller (7)-(8) to the system (1) to show its validity. To this end, we let $K=[-1,-3,-3]$ where $A_K$ is Hurwitz with $\sigma=0.1, \epsilon=0.1$. and $T=0.001$. From [1], the nominal system parameters are as follows: $Bm=2.68042\times10^{-5}Nms$, $K_b=0.0603V_s$, $K_m=0.060438586Nm/A$, $R=1.16\Omega$, $J_m=0.0000134Kgm^2$, and $r=1$. For comparison purposes, we compare our proposed method to the time-triggered controller of [1] in tracking a step signal. The simulation results are shown in Fig. 1 and Table 1 where we can observe that the output trajectories are similar while the proposed event-triggered controller uses significantly reduced control input updates.




(a) Output trajectories comparison.




(b) Interexecution times of the proposed event-triggered controller.

그림 1. 시뮬레이션 결과.

Fig. 1. Simulation results.

표 1. 제어 입력 업데이트 수 비교.

Table 1. Comparison of the number of control input updates.

| | Time-triggered PID control [1] | Event-triggered PID control |
|---|---|---|
| Number of control input updates | 10 , 000 | 352 |

## V. CONCLUSIONS

We have proposed an event-triggered ϵ-PID controller for the position control of a DC motor. We have newly designed a triggering condition to cope with the event-triggering mechanism and presented a system analysis to show that the controlled system is well bounded. Our proposed controller retains the intrinsic merit of the event-triggered control method, that is the number of control input updates is significantly reduced. Via simulation results, we have shown that our proposed controller yields the similar control performance to the time-triggered controller while using much less control input updates.

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